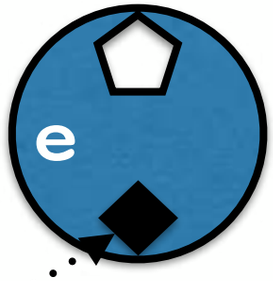
Noninterference for Free

William J. Bowman and Amal Ahmed

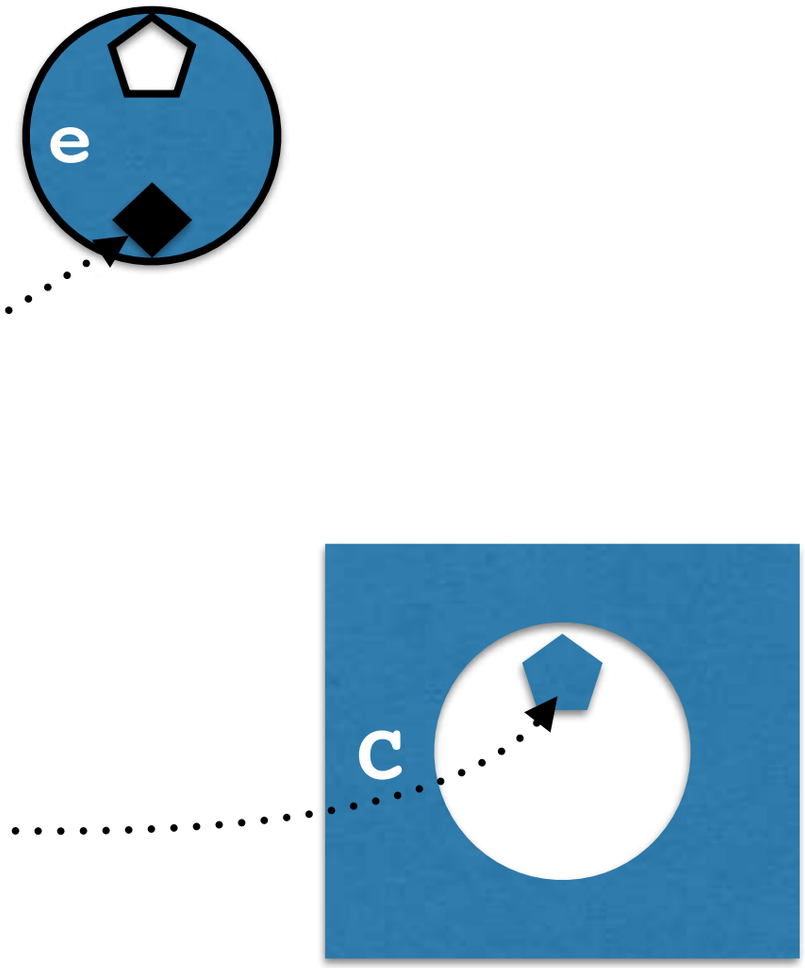
Let’s write a secure program

- Want to write a component `e` (browser)
- Manages high-security data (passwords)

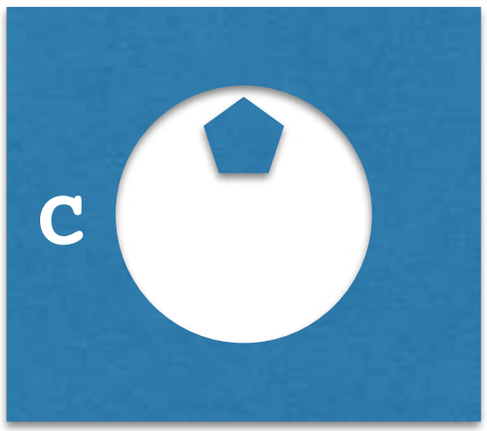


Let's write a secure program

- Want to write a component **e** (browser)
- Manages high-security data (passwords)
- Links with untrusted context **c** (plugins)
- **c** provides low-security inputs, reads low-security outputs

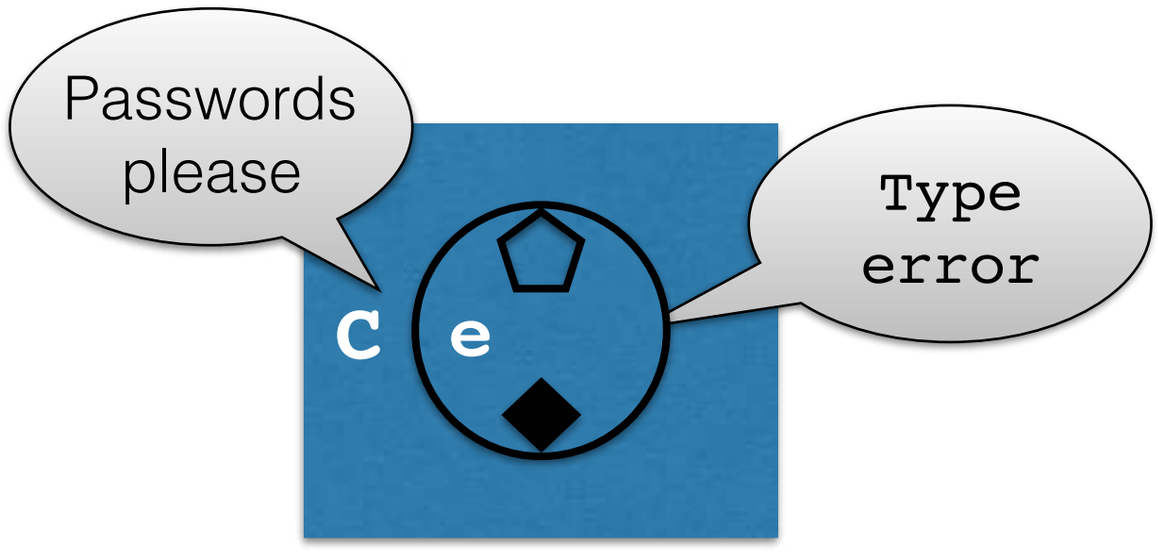


Language-based security!



Using language-based security,
we statically rule out attacks

Language-based security!



Using language-based security,
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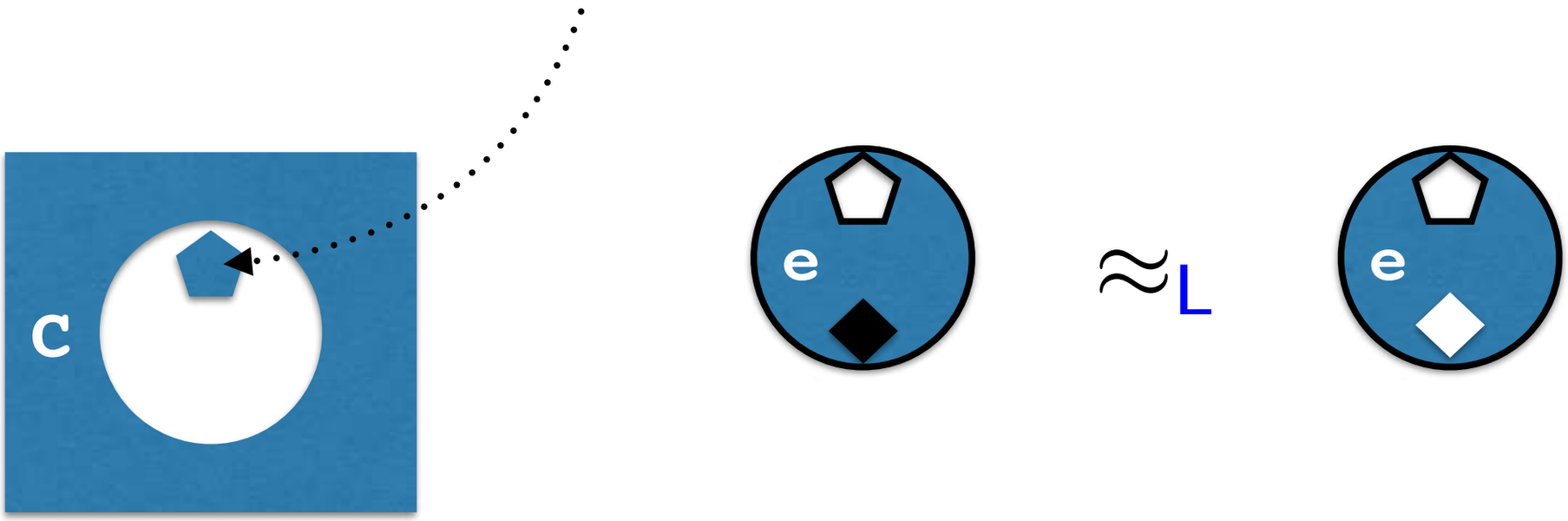
Noninterference

Noninterference is an **equivalence property** of any well-typed term `e`:

Given ***same*** low-level (***public***) inputs,



Noninterference

Noninterference is an **equivalence property**
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Given ***same*** low-level (***public***) inputs,

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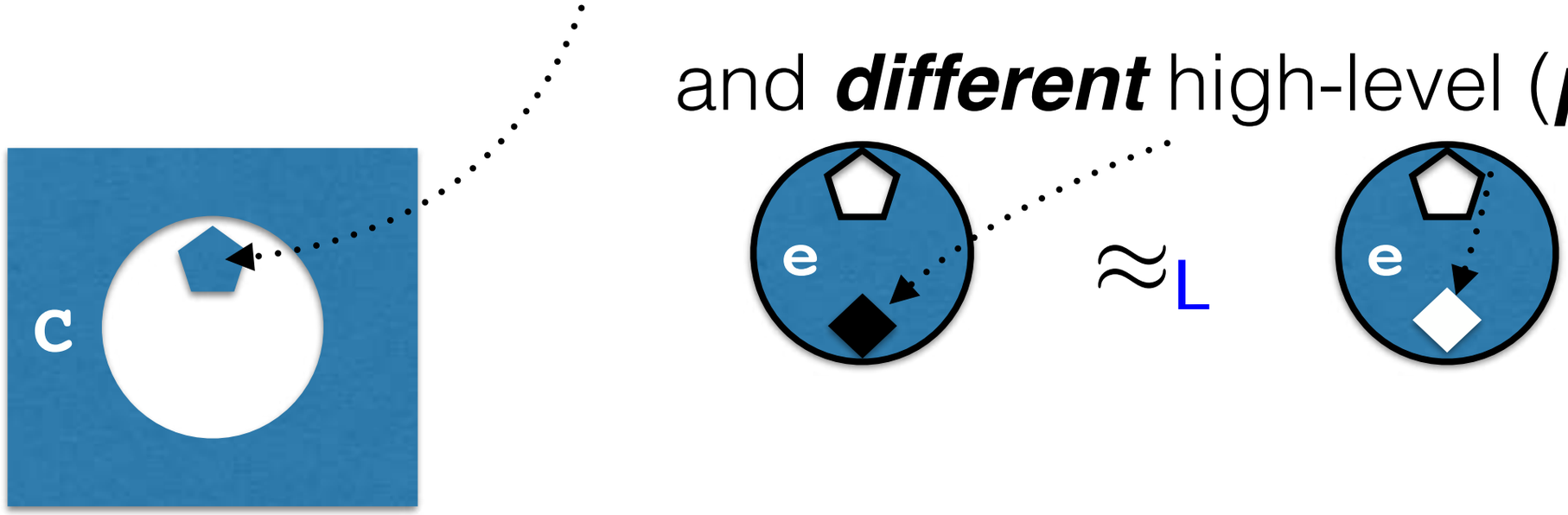

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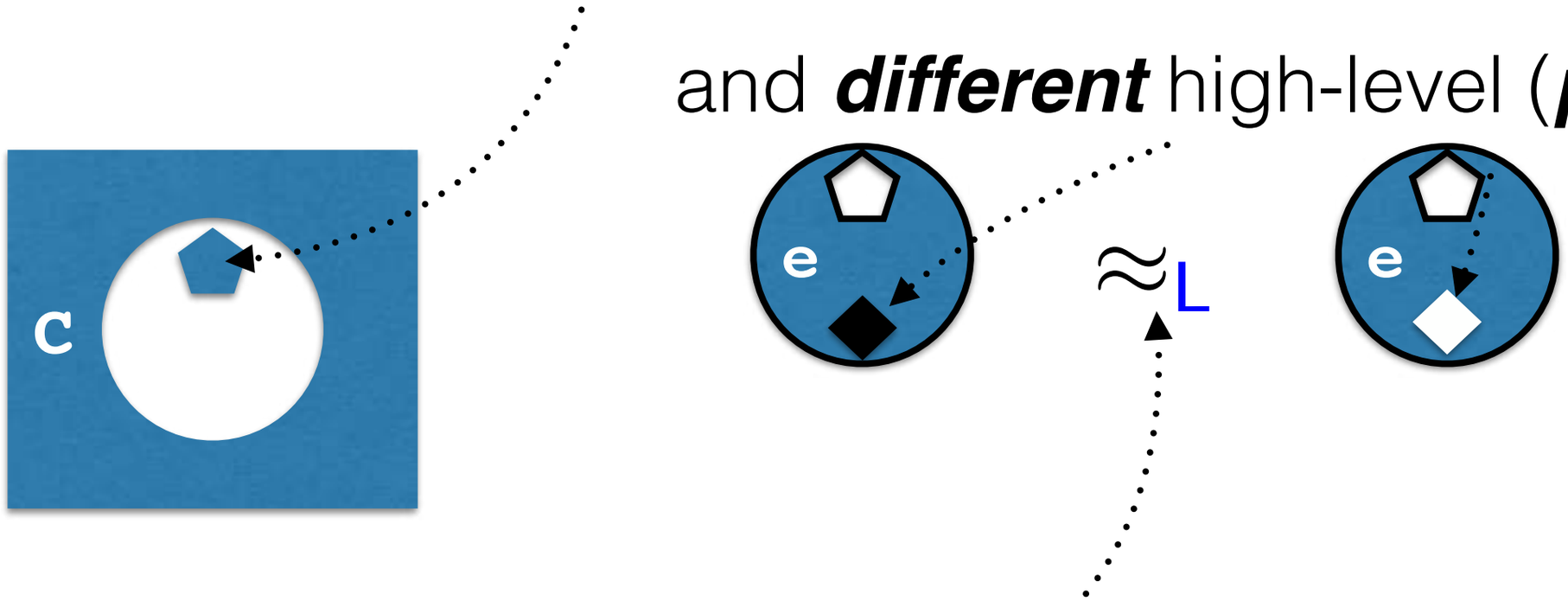


low-level outputs are indistinguishable

Noninterference

Noninterference is an **equivalence property**

of any well-typed term c:

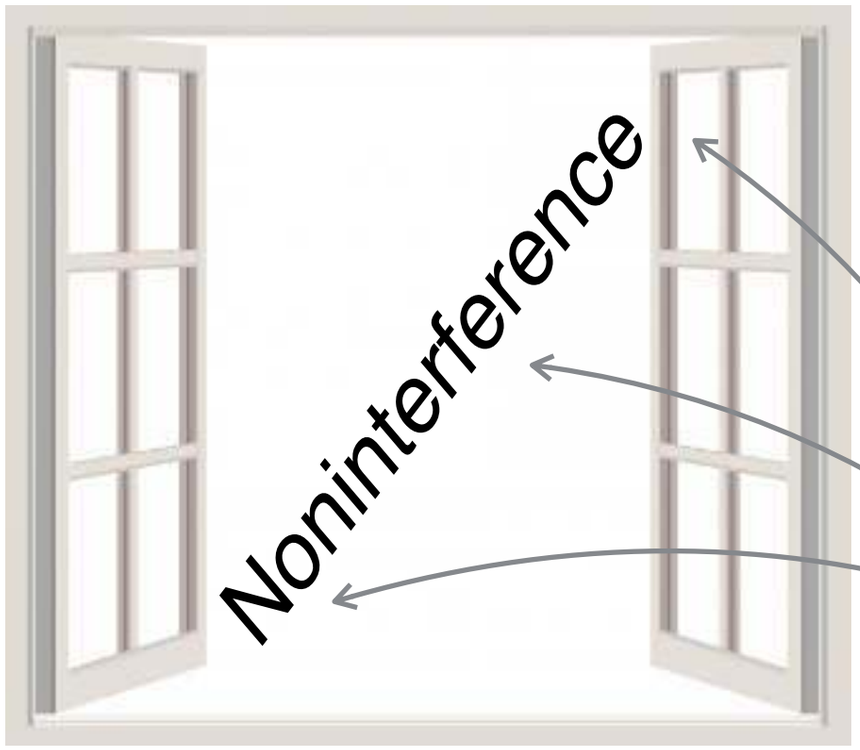
Given ***same*** ***vate***) inputs

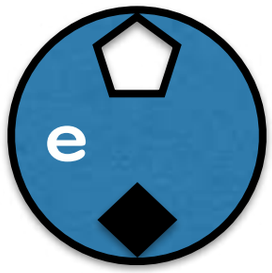
Security Solved!

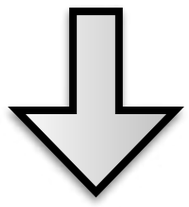
low-level outputs are indistinguishable

WRONG

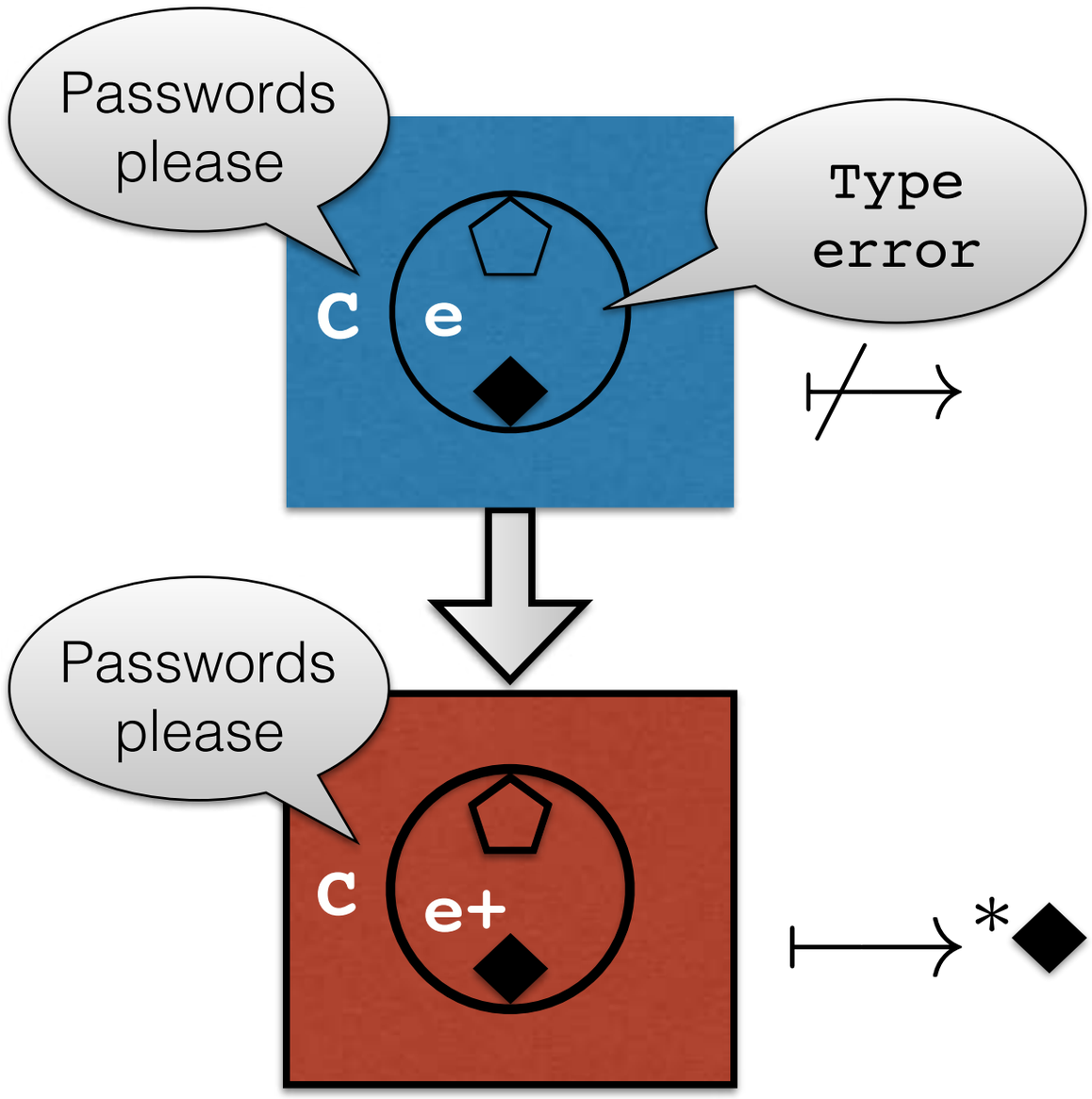
Because compilers





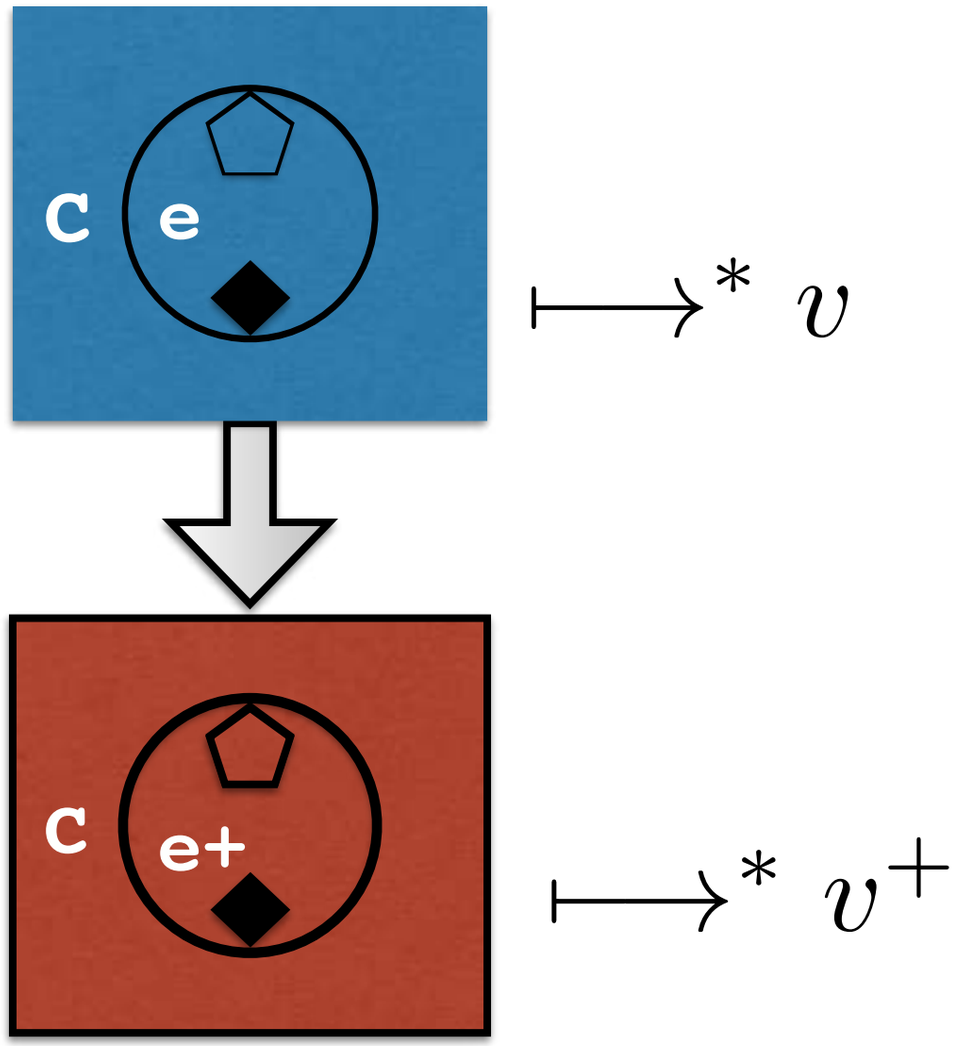


Because compilers



“Correct”

Even if the compiler is proven “correct”…

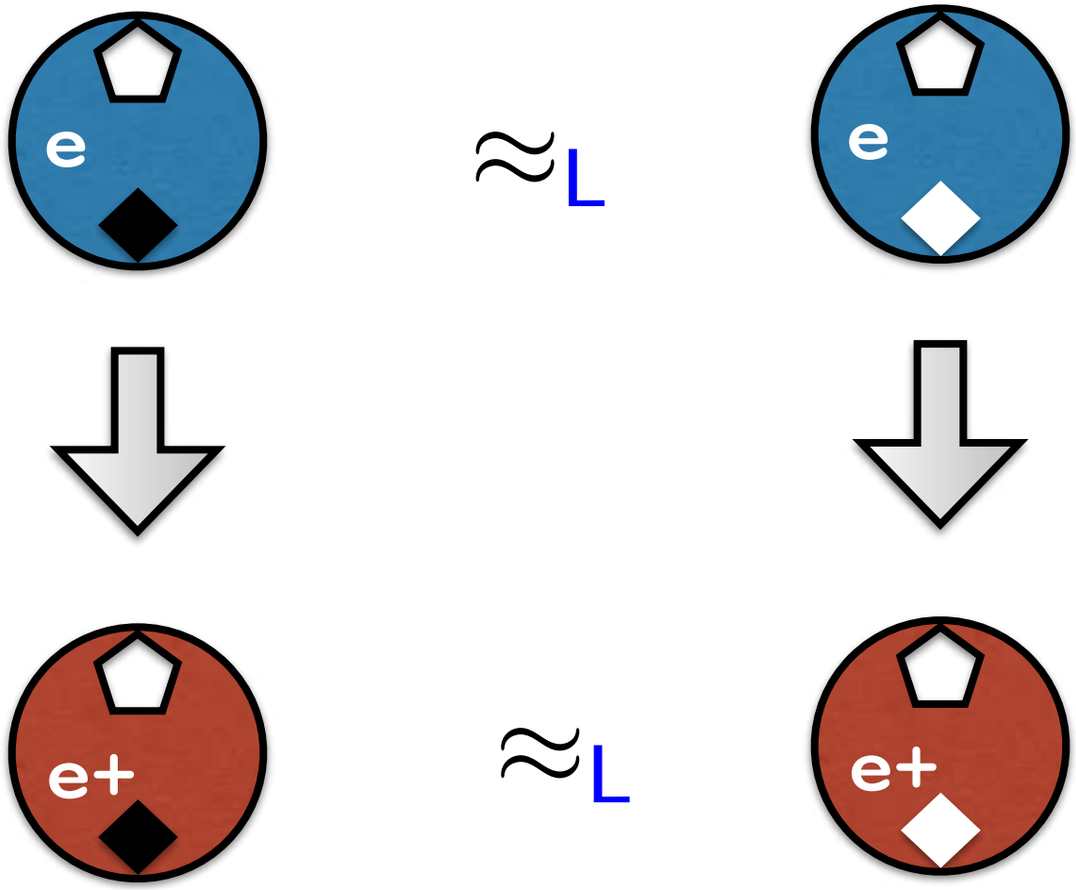


$$\longmapsto^* v$$

$$\longmapsto^* v^+$$

Equivalence Preserving

It may not preserve equivalences, e.g., noninterference.



How do we preserve noninterference?

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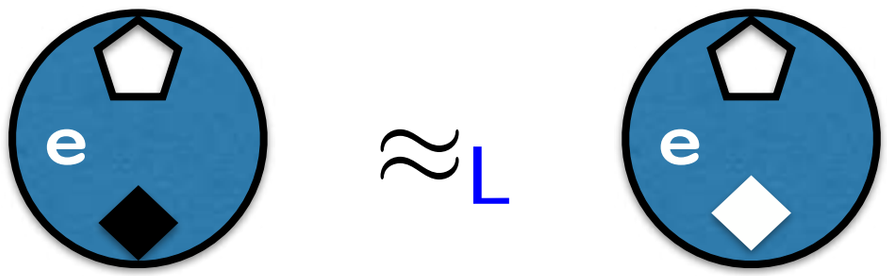
Folklore suggests noninterference can be



captured by parametricity

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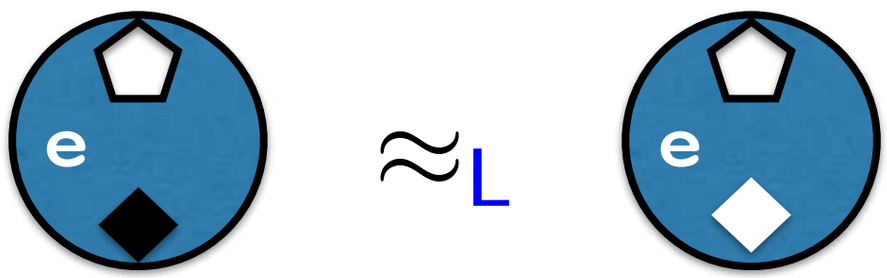


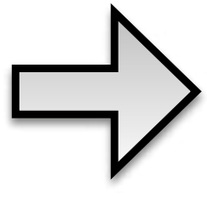
captured by parametricity

[1] Tse & Zdancewic, *Translating Dependency into Parametricity*, ICFP 2004

[2] Shikuma & Igarashi, *Proving Noninterference by a Fully Complete …*, ASIAN 2006

Languages

Source: DCC [1]



Target: System Fω

- Captures dependency analyses
  - e.g. Information-flow security
- STLC + Lattice of monads

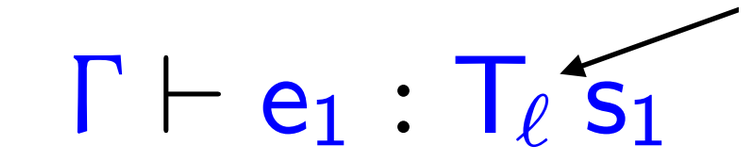
- Parametricity
- Type constructors, i.e., higher-order polymorphism

[1] Abadi *et al*., *The Core Calculus of Dependency,* POPL 1999

Source Language

DCC (Core Calculus of Dependency)

Monad protects
data based on label



Source Language

DCC (Core Calculus of Dependency)

$\eta_{\mathsf{H}}\ \mathsf{true} \approx_{\mathsf{L}} \eta_{\mathsf{H}}\ \mathsf{false} : \mathsf{T}_{\mathsf{H}}\ \mathsf{bool}$

Monad protects data based on label

$\Gamma \vdash \mathsf{e}_1 : \mathsf{T}_{\ell}\ \mathsf{s}_1$

Source Language

DCC (Core Calculus of Dependency)

$$\frac{\Gamma \vdash e_1 : T_\ell\ s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2}{\Gamma \vdash \mathsf{bind}\ x = e_1\ \mathsf{in}\ e_2 : s_2}$$

Term containing private data

Continuation using private data

Source Language

DCC (Core Calculus of Dependency)

Promise that result is protected

$$\frac{\Gamma \vdash e_1 : T_\ell\ s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2 \qquad \ell \preceq s_2}{\Gamma \vdash \mathsf{bind}\ x = e_1\ \mathsf{in}\ e_2 : s_2}$$

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DCC (Core Calculus of Dependency)

Promise that result is protected

For example…

$L \not\preceq \text{bool}$

$H \preceq 1$

$\ell \preceq s_2$

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$L \not\preceq \text{bool}$

$L \preceq T_L\ s$

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Source Language

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$L \not\preceq \mathsf{bool}$	$H \preceq 1$
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$L \preceq T_H\ s$	$H \preceq T_H\ s$

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Term containing private data

Continuation using private data

Translation, Briefly

How are types translated?

$$1^+ = \mathbf{1}$$

$$\mathsf{bool}^+ = \mathbf{bool}$$

$$(\mathsf{s}_1 \to \mathsf{s}_2)^+ = \mathsf{s}_1^+ \to \mathsf{s}_2^+$$

$$(\mathsf{T}_\ell\ \mathsf{s})^+ = \ ?$$

Translation, Briefly

$$\frac{\Gamma \vdash e_1 : T_\ell\ s_1 \qquad \Gamma, x : s_1 \vdash e_2 : s_2 \qquad \ell \preceq s_2}{\Gamma \vdash \mathsf{bind}\ x = e_1\ \mathsf{in}\ e_2 : s_2}$$

Idea: CPS the monad + constrain continuation result

$$(T_\ell\ s)^+ = \forall \beta{::}*.\ (s^+ \to \beta) \to \beta$$

s.t. $\ell \preceq \beta$

Translation, Briefly

$$\frac{\Gamma \vdash \mathsf{e}_1 : \mathsf{T}_\ell\ \mathsf{s}_1 \qquad \Gamma, \mathsf{x} : \mathsf{s}_1 \vdash \mathsf{e}_2 : \mathsf{s}_2 \qquad \ell \preceq \mathsf{s}_2}{\Gamma \vdash \mathsf{bind}\ \mathsf{x} = \mathsf{e}_1\ \mathsf{in}\ \mathsf{e}_2 : \mathsf{s}_2}$$

$$(\mathsf{T}_\ell\ \mathsf{s})^+ = \forall \beta{::}*.\ ([\![\ell \preceq \beta]\!] \times (\mathsf{s}^+ \to \beta)) \to \beta$$

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$$[\![\ell \preceq \mathsf{s}^+]\!] = (\alpha_{\preceq}\ \ \alpha_\ell\ \ \mathsf{s}^+)$$

Translation, Briefly

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$$\begin{aligned}
&\mathtt{data}\ (\boldsymbol{\alpha}_{\preceq}\ \ \boldsymbol{\alpha}_\ell\ \ \mathsf{s}^+)\ \mathtt{where} \\
&\quad \mathtt{Unit_P} :: (\boldsymbol{\alpha}_{\preceq}\ \ \boldsymbol{\alpha}_\ell\ \ 1) \\
&\quad \ldots \\
&\quad \mathtt{Monad_P} :: [\![\ell \sqsubseteq \ell']\!] \to (\boldsymbol{\alpha}_{\preceq}\ \ \boldsymbol{\alpha}_\ell\ \ (\mathsf{T}_{\ell'}\ \mathsf{s})^+)
\end{aligned}$$

Translation Summary

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Invariants:

$$\vdash \mathsf{s} \qquad\qquad \boldsymbol{\alpha}_{\underline{\preceq}}, \boldsymbol{\alpha}_\ell, \cdots \vdash \mathsf{s}^+$$

Proving Noninterference Preservation

Proving Equivalence

Preservation

Equiv. Preservation is Hard

To *show*

$$\lambda x\!:\!s.\, e \approx \lambda x\!:\!s.\, e'$$

implies

$$\boldsymbol{\lambda x}\!:\!s^{+}.e^{+} \approx \boldsymbol{\lambda x}\!:\!s^{+}.e'^{+}$$

Equiv. Preservation is Hard

To *show*

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implies

$$\lambda x{:}s^{+}.e^{+} \approx \lambda x{:}s^{+}.e'^{+}$$

Assume $\quad \mathbf{m_1} \approx \mathbf{m_2} : s^{+}$

Show $\quad e^{+}[\mathbf{m_1}/\mathbf{x}] \approx e'^{+}[\mathbf{m_2}/\mathbf{x}]$

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Want to proceed as follows:

- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$

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Want to proceed as follows:

- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$
- Hence, $e[\mathbf{m_1}/x] \approx e'[\mathbf{m_2}/x]$
- By induction

Equivalence Reflection?

How do we say that since $\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$

$\wr \quad \wr$

there must exist $\mathsf{e}_1 \quad \mathsf{e}_2 : \mathsf{s}$

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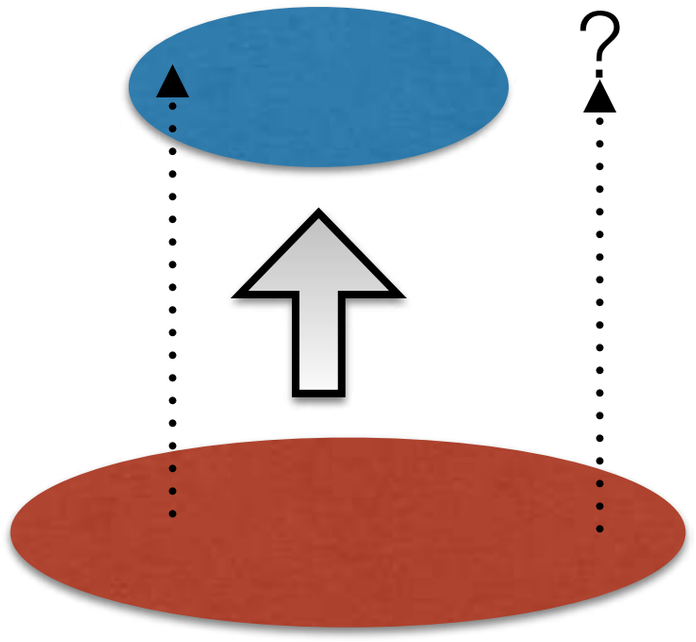
Back-translation

How can we possibly back-translate
a *more expressive* target language?



Enrich Source?

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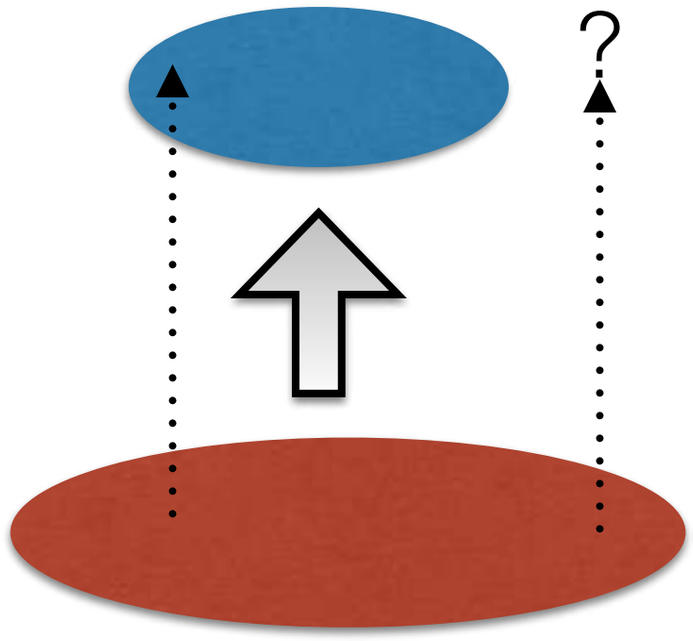

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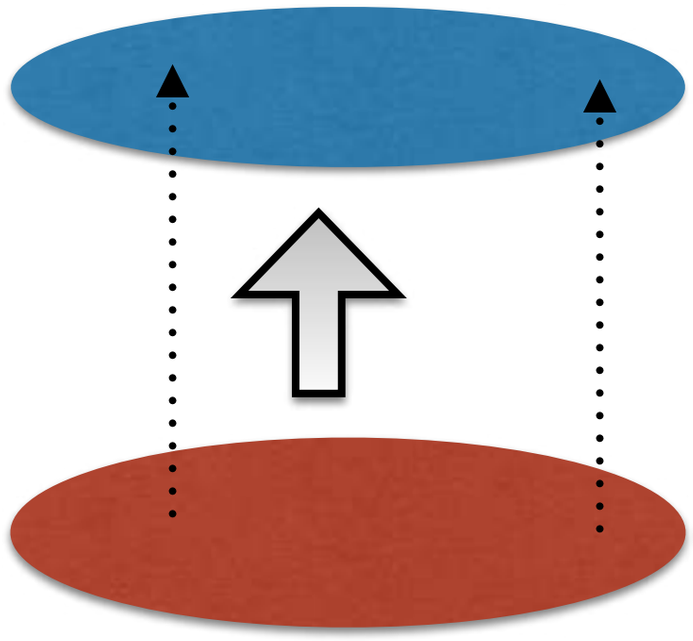

We could enrich the source

Impoverish Target?

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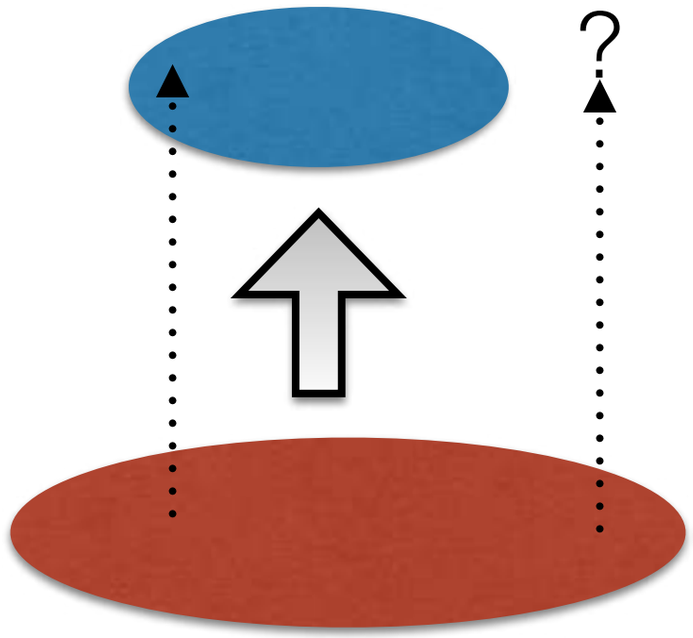

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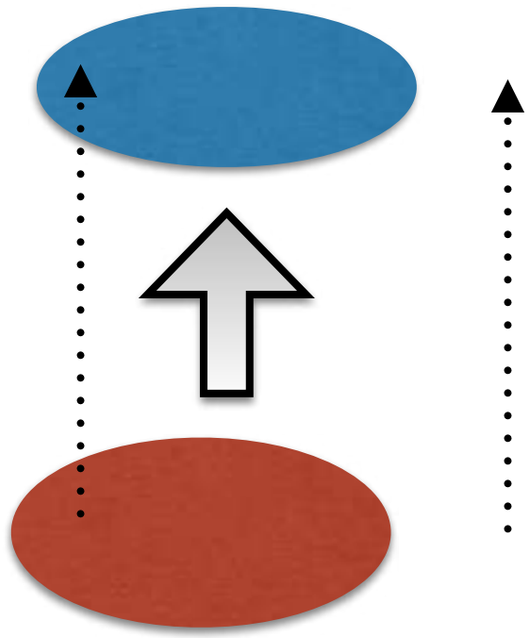

Or impoverish the target

None of the above

How can we possibly back-translate
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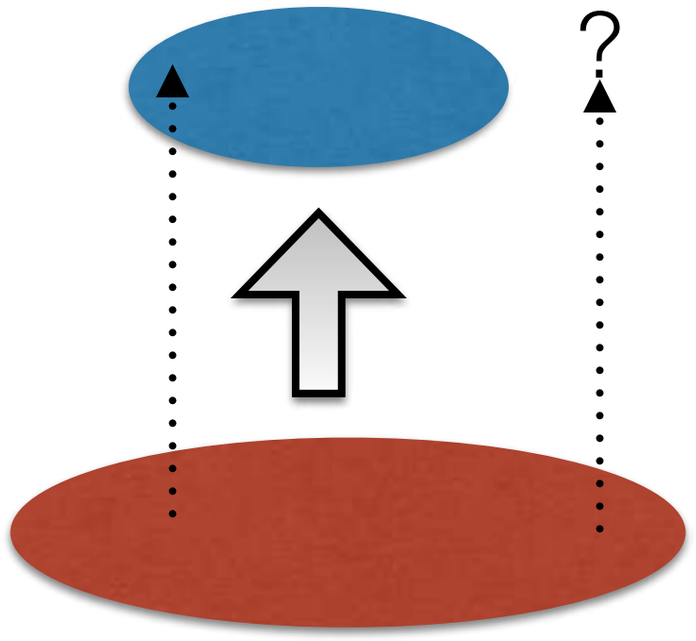
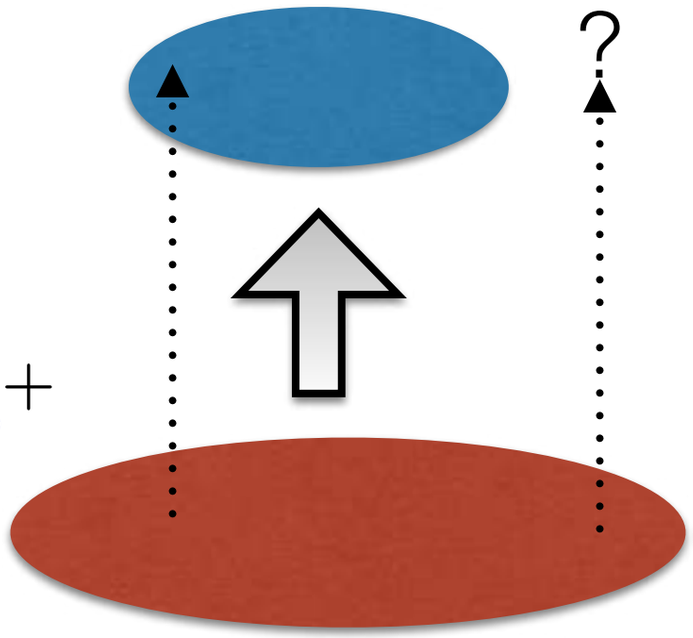
Neither is satisfying



Be clever

How can we possibly back-translate a *more expressive* target language?



Recall:

Assume $\mathbf{m_1} \approx \mathbf{m_2} : \mathbf{s}^+$

Be clever

How can we possibly back-translate
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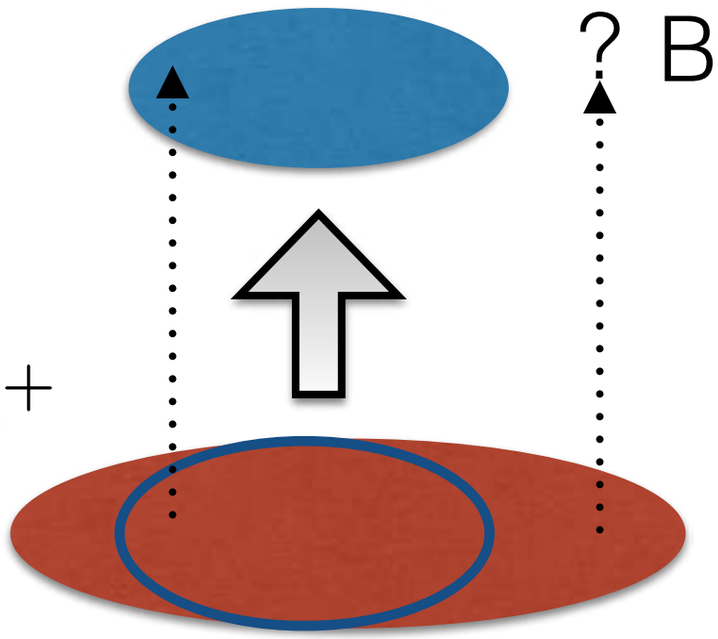
? By using types.



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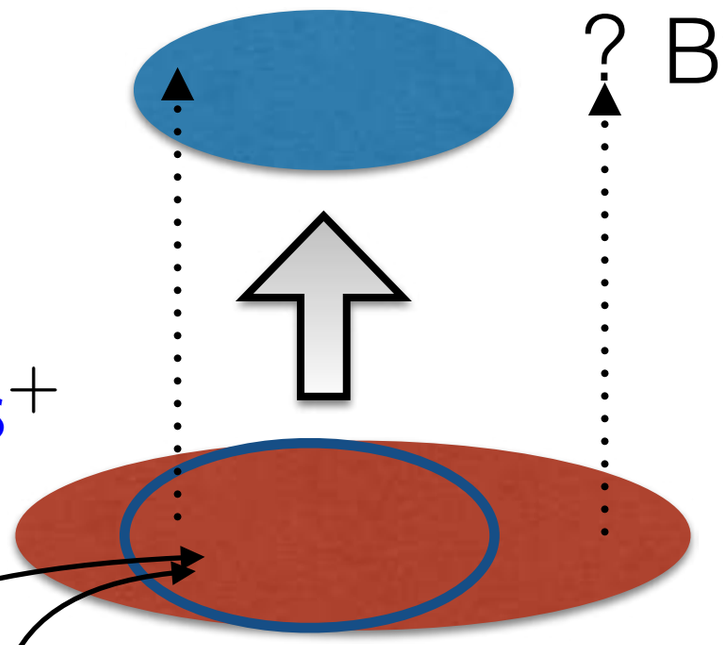

e.g.

$$\mathbf{x} : s_1^+ \vdash \mathbf{m} : s_2^+ \quad \cdots\cdots\blacktriangleright \quad x : s_1 \vdash e : s_2$$

$$\boldsymbol{\lambda x{:}s_1^+ .\, m} : s_1^+ \to s_2^+ \quad \cdots\cdots\blacktriangleright \quad \lambda x : s_1 .\, e : s_1 \to s_2$$

Be clever

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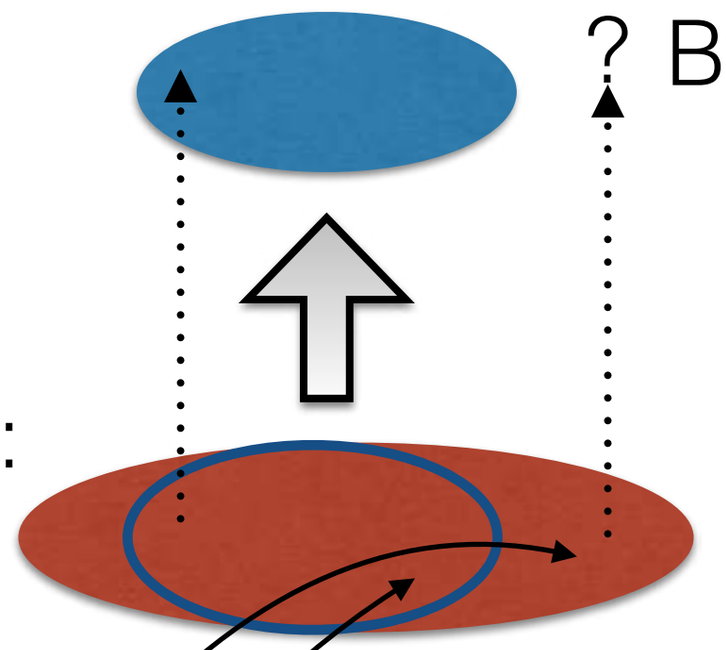

Can we back-translate:

$\mathbf{m'} : \mathbf{t}$

$(\mathbf{\lambda x{:}t.m})\ \mathbf{m'} : \mathtt{s}^{+}$

Non-translation type sub-term

Translation type

Be cleverer

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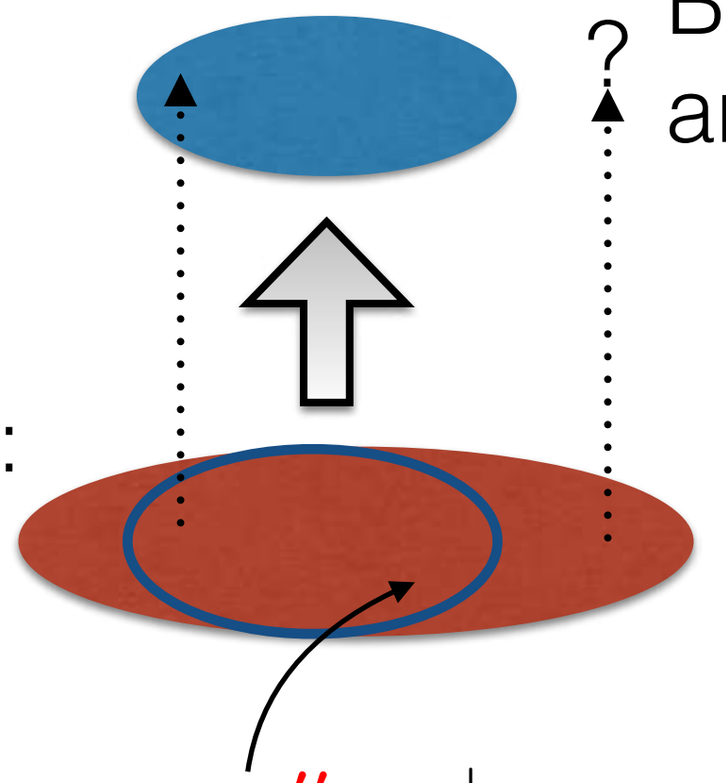

By using types
and partial evaluation.

Can we back-translate:

Not by induction

$(\lambda x{:}t.m)\ m' \longmapsto m'' : s^+$

$m' : t$ is no more

Be clevererer

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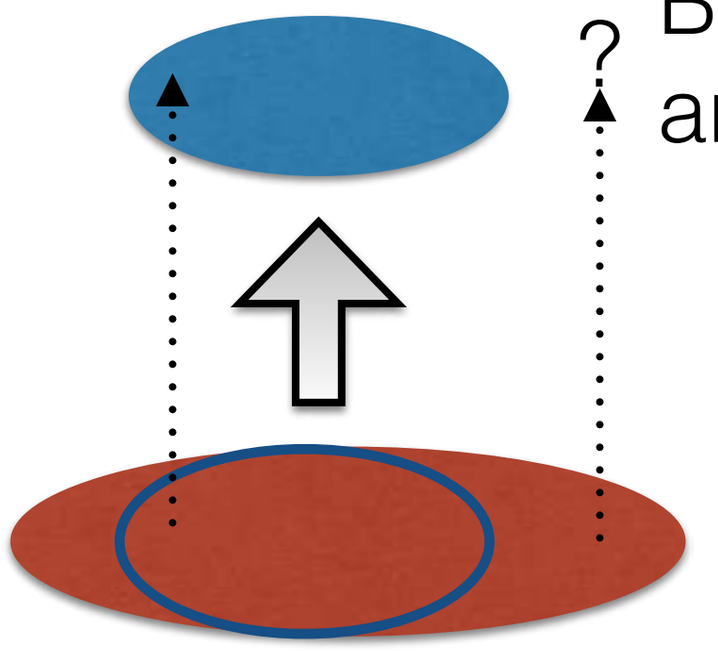

By using types
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Not by induction

Therefore, must prove all terms are back-translatable.

Proof of Equiv. Preservation

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\lambda x{:}s^+ .e^+ \approx \lambda x{:}s^+ .e'^+$$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

How the proof proceeds:

Proof of Equiv. Preservation

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\boldsymbol{\lambda x{:}}s^+ \boldsymbol{.} e^+ \approx \boldsymbol{\lambda x{:}}s^+ \boldsymbol{.} e'^+$$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

How the proof proceeds:

- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^+$ to $e_1 \approx e_2 : s$

Proof of Equiv. Preservation

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\boldsymbol{\lambda x{:}s^{+}}.e^{+} \approx \boldsymbol{\lambda x{:}s^{+}}.e'^{+}$$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$

Show $e^{+}[\mathbf{m_1}/\mathbf{x}] \approx e'^{+}[\mathbf{m_2}/\mathbf{x}]$

How the proof proceeds:

- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$ to $e_1 \approx e_2 : s$
- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$

Proof of Equiv. Preservation

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\boldsymbol{\lambda x{:}s^{+}.e^{+}} \approx \boldsymbol{\lambda x{:}s^{+}.e'^{+}}$$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$

Show $e^{+}[\mathbf{m_1}/\mathbf{x}] \approx e'^{+}[\mathbf{m_2}/\mathbf{x}]$

How the proof proceeds:

- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^{+}$ to $e_1 \approx e_2 : s$
- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$
- Hence, $e[e_1/x] \approx e'[e_2/x]$

Proof of Equiv. Preservation

To *show*

$$\lambda x : s.\, e \approx \lambda x : s.\, e'$$

implies

$$\lambda x{:}s^+ .e^+ \approx \lambda x{:}s^+ .e'^+$$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

How the proof proceeds:

- Back-translate $\mathbf{m_1} \approx \mathbf{m_2} : s^+$ to $e_1 \approx e_2 : s$
- By assumption, $\lambda x : s.\, e \approx \lambda x : s.\, e'$
- Hence, $e[e_1/x] \approx e'[e_2/x]$
- By induction. QED.

Proof of Equiv. Preservation

To *show*

$\lambda x : s.\, e \approx \lambda x : s.\, e'$

implies

$\lambda x{:}s^+ .e^+ \approx \lambda x$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

EXCEPT

roceeds:

$_1 \approx \mathbf{m_2} : s^+$

$x : s.\, e \approx \lambda x : s.\, e'$

$e'[e_2/x]$

.

Proof of Equiv. Preservation

To *show*

$\lambda x : s.\ e \approx \lambda x : s.\ e'$

implies

$\lambda x{:}s^+ .e^+ \approx \lambda x$

Assume $\mathbf{m_1} \approx \mathbf{m_2} : s^+$

Show $e^+[\mathbf{m_1}/\mathbf{x}] \approx e'^+[\mathbf{m_2}/\mathbf{x}]$

EXCEPT

oes:

$_1 \approx \mathbf{m_2} : s^+$

$x : s.\ e \approx \lambda x : s.\ e'$

$e'[e_2/x]$

.

If you don't understand logical relations, you can stop listening for the next 4 slides.

“Open” Logical Relations

Typically, logical relations are defined on closed terms/types.

Again recall: Assume $\mathbf{m_1} \approx \mathbf{m_2} : \mathbf{s}^+$

And: $\boldsymbol{\alpha}_{\underline{\prec}}, \boldsymbol{\alpha}_{\ell}, \cdots \vdash \mathbf{s}^+$

But translation types are only well-formed when open.

"Open" Logical Relations

Again recall: Assume $\mathbf{m_1} \approx \mathbf{m_2} : \mathbf{s}^+$

And: $\boldsymbol{\alpha} \underline{\preceq}, \boldsymbol{\alpha}_\ell, \cdots \vdash \mathbf{s}^+$

i.e., to even *state* this assumption, the logical relation *must* leave these type variables open.

"Open" Logical Relations

~~$\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$~~

building on [1], we define $\mathbf{m_1} \approx^{\mathbf{\Sigma}} \mathbf{m_2} : \mathsf{s}^+$

$$\mathbf{\Sigma} = \begin{array}{l} \mathbf{data}\ (\boldsymbol{\alpha}_{\preceq}\ \ \boldsymbol{\alpha}_{\ell}\ \ \mathsf{s}^+)\ \mathbf{where} \\ \quad \mathbf{Unit\_P} :: (\boldsymbol{\alpha}_{\preceq}\ \ \boldsymbol{\alpha}_{\ell}\ \ 1) \\ \quad \ldots \\ \quad \mathbf{Monad\_P} :: (\boldsymbol{\alpha}_{\preceq}\ \ \boldsymbol{\alpha}_{\ell}\ \ (\mathsf{T}_{\ell}\,\mathsf{s})^+) \end{array}$$

[1] Zhao *et al.*, *Relational Parametricity for Linear System F*, APLAS 2010

QED! (ish)

~~$\mathbf{m_1} \approx \mathbf{m_2} : \mathsf{s}^+$~~

$\mathbf{m_1} \approx^{\mathbf{\Sigma}} \mathbf{m_2} : \mathsf{s}^+$

$\mathbf{\Sigma} =$

```
data (α≼ αℓ s+) where
  Unit_P :: (α≼ αℓ 1)
  ...
  Monad_P :: (α≼ αℓ (Tℓ s)+)
```

$[\![\mathbf{\Sigma}]\!]_{\mathsf{L}} = ?$

Conclusion

- Language-based reasoning requires better compilers
- We have developed techniques for such compilers (specifically, for noninterference preservation)

https://www.williamjbowman.com/papers#niforfree